Enhancing the Reconstruction of Acoustic Source Field Using Wavelet Transformation

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This paper shows the use of wavelet transformation combined with inverse acoustics to reconstruct the surface velocity of a noise source. This approach uses the boundary element analysis based on the measured sound pressure at a set of field points, the Helmholtz integral equations and wavelet transformation for reconstructing the normal surface velocity field. The reconstructed field can be diverged due to the small measurement errors in the case of nearfield acoustic holography (NAH) using an inverse boundary element method. In order to avoid this instability in the inverse problem, the reconstruction process should include some form of regularization for enhancing the resolution of source images. The usual method of regularization has been the truncation of wave vectors associated with small singular values, although the order of an optimal truncation is difficult to determine. In this paper, a wavelet transformation is applied to reduce the computation time for inverse acoustics and to enhance the reconstructed vibration field. The computational speed-up is achieved, with solution time being reduced to 14.5%.

Key Words : Wavelet, Wavelet Transformation, Boundary Element Method (BEM), Nearfield Acoustic Holography (NAH), Acoustics

1. Introduction

In practical acoustic engineering applications, it is often required to reconstruct vibration field as an inverse problem so as to come up with better designs to reduce resulting structure-borne noise. In case of an inverse acoustic radiation problem, it is to determine the acoustic quantities on the source surface based on measured acoustic pressure signals in the field.

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There had been many investigations for identifying or visualizing normal velocity field using inverse technique. Identification of velocity field is very important because one can control the noise after finding vibration and/or acoustic sources in the system such as a vehicle or an aircraft. Acoustic holography method is one of the best schemes for identifying and visualizing an inverse acoustic field. The quality of the hologram depends strongly upon the size of the hologram surface, its microphone spacing, the spatial resolution, errors including the incorrect position of the microphones. Errors appeared on the hologram surface significantly affect the quality of the vibration or acoustic field that will be reconstructed on the source surface.

Identification of the acoustic field was initiated from imaging schemes formulated as optical holography equations, which can be found in Goodman (1968). Nearfield acoustic holography (NAH) can be used to get the source position at the surface, or to determine how the sound field propagates from the surface and into the farfield. The advantages of NAH include the consistent spatial data sets, the possibility of an automated measurement process, the evaluation of transient or steady state noise sources, and the availability of 3D acoustic quantities. Inverse acoustics using NAH, which is proposed by Maynard (1985), Veronesi et al. (1987) and Wang (1997), has been the subject of extensive studies for the past two decades and has been investigated in many areas by several researchers like Maynard (1985), Veronesi et al. (1987), Wang and Wu (1997), Dumbacher (1999), Borgiotti (1990).

Inverse acoustics using a boundary element method (BEM) based acoustical holography has been suggested with similar concepts of the conventional NAH based on the spatial Fourier transform by Bai (1992) and Kim and Ih (1996). This technique has the following advantages compared with conventional NAH: the pressure need not be measured in separable coordinates, thus a reduced number of measurements with uneven spacing is possible; reflections from all directions can be considered; concave regions of the source can be reconstructed; and wrap-around error due to the finite aperture size is not involved. Due to these merits, the actual reconstruction of vibration fields using this technique has been performed by Veronesi et al. (1989), Gardner et al. (1988), Ih et al. (1998) and Williams (1999).

It is well known that the application of the boundary element method (BEM), proposed by Brebbia (1984), for the numerical solution of boundary-value problems produces non-symmetric, dense matrices, which are computationally expensive to solve using direct methods such as Gauss elimination, requiring an effort of order $O(N3)$ for systems with N degrees of freedom. To reduce the computational effort involved in the solution, several papers have been published on iterative methods such as conjugate gradients, Lanczos or GMRES proposed by Youcef et al. (1986), Barra et al. (1994), Kane et al. (1991) and Mansur et al. (1992). Despite some relative success, the time necessary to achieve accurate solutions, even with well-tuned iterative solvers, is still of order $O(N2)$. There is a great effort, at present, on the development of very fast solvers for the BEM, of order $O(N \log N)$, based on the idea of reducing the redundancy by using panel clustering (Hackbusch et al., 1989) or fast multipole expansions (Rokhlin, 1983; Nabors et al., 1994; Gyure et al., 1998).

However, these techniques require completely new numerical formulations and related computer programs. Another alternative scheme in this paper is to apply wavelet transforms. The main advantage of this technique is that it can be directly used as a post processor in existing BEM programs. The application of wavelet compression techniques for the solution of BEM systems of equations is still in its infancy and only very simple problems of potential theory have been dealt with so far (Beylkin et al., 1991; Bond et al., 1994; Gonzalez et al., 2000). Robust wavelet compression algorithms are necessary if the technique is to be applied for the solution of practical engineering problems. The present work describes the implementation of a wavelet-based solver and discusses several particular features resulting from the application of wavelet transform techniques to the boundary element method.

Wavelet analysis was initiated by Haar (1910) and the concept of wavelets in theoretical form was first proposed by Morlet (1983). The methods of wavelet analysis have been mainly developed by Meyer (1993). The wavelet transform has become a standard tool in signal and image processing, and it has found applications to almost all fields of physics, engineering and applied mathematics. There are many practical applications for de-noising including image, video quality enhancement and even astronomy and astrophysics for studying solar corona and detection of gamma sources. Said and Peralman (1996), Shapiro (1993) reported that the discrete wavelet transform is appropriate for signal compression and reconstruction, and it is especially efficient in the signal processing community. De-noising using the wavelet is used to produce enhanced estimates of an image corrupted by noise. The restored image should contain less noise than the original noisy one and be sharp. Chambolle (1998), Chang and Vetterli (1997), Coifman and Donoho (1995), Donoho (1992, 1994, 1995), Shao and Cherkassky (1998) reported that it means that de-noising should result in sharpening the edges of the original noisy image. One of the key properties underlying the success of wavelets is that the wavelet transform provides excellent localization in both time and frequency domain. In addition, wavelet expansions tend to concentrate the signal energy within a relatively small number of (large) coefficients.

Simoncelli (1999) reported de-noising can be viewed as a signal estimation problem in which one wants to estimate the original (noise-free) image signal from the noisy samples. The wavelet thresholding method originally proposed by Donoho (1992, 1994, 1995), is a signal estimation technique that exploits remarkable abilities of wavelet transform for signal de-noising and compression. It removes noise by discarding coefficients that are insignificant relative to some threshold, assuming that the small coefficients are mainly contributed by the additive noise. Coifman and Donoho (1995), Donoho (1994, 1995) reported that wavelet thresholding solution is asymptotically optimal in a minimax mean squared error (MSE) sense over a variety of smoothness spaces.

This paper is mainly concerned with an inverse acoustics using NAH and wavelet analysis. The surface velocity field on the source surface is reconstructed from measured acoustic pressure signals on a hologram surface. This paper proposes a method to get the enhanced source field from a noisy one by combining a wavelet transformation with NAH. The proposed method obtains the de-noised field from a noisy field on a source plane reconstructed through backward propagation of NAH.

2. Inverse Acoustics Using BEM

Many investigations of acoustics using BEM are investigated by Kirkup (1994, 1998), Bernard et al.(1987), Ih (1996) and other researchers. The linear algebraic equations can be obtained by the use of interpolation and the numerical integration of the well-known Kirchhoff-Helmholtz boundary integral equation as follows:

$$[G]\left[\frac{\partial p}{\partial n}\right]_s=[F][f]_s \tag{1}$$

where subscript s means the source surface. Eq. (1) is then manipulated to solve for the boundary quantities. Once the surface acoustic quantities are known, the Kirchhoff-Helmholtz equation can be used to solve for the pressure at any location within the problem domain. The pressure at a set of field points, p_f, is then given by

$$p_f=[G']\left[\frac{\partial p}{\partial n}\right]_s-[F'][p]_s \tag{2}$$

In NAH, a set of measured acoustic pressures is back propagated to the surface of the source to predict the pressure and velocity variables on the surface. Once the surface vibrations are identified, then any acoustical quantity in the sound field may be calculated. All of the relationships necessary to relate the surface velocities on a source surface to the measured sound pressures in the radiated sound field are available within the boundary element equations. The surface velocities, $\frac{\partial p}{\partial n}$, are related to field pressures, p_f, by

$$\begin{aligned} p_f&=[[G']-[F'][F]^{-1}[G]]\left[\frac{\partial p}{\partial n}\right]_s \\ &=[G'']\left[\frac{\partial p}{\partial n}\right]_s \end{aligned} \tag{3}$$

where $[G'']$ is the propagation matrix that may be used to calculate the field pressure from surface velocities.

The propagation matrix can be inverted by using singular value decomposition (SVD) algorithm. Thus the back-propagation algorithm for the boundary element implementation of NAH is

$$\left[\frac{\partial p}{\partial n}\right]_s=[G'']^H p_f \tag{4}$$

where $[G'']^H$ is a pseudo-inverse of the propagation matrix. Therefore the surface velocity field can be obtained from the following equation.

$$\begin{aligned}\left[\frac{\partial p}{\partial n}\right]_s &= [G'']^H p_f = (U\Sigma V^H)^H p_f \\ &= V\Sigma U^H p_f\end{aligned} \tag{5}$$

where superscript H denotes the Hermitian operator. The singular value decomposition for $[G'']$ is given by

$$[G''] = U\Sigma V^H \tag{6}$$

and

$$\Sigma = diag(\Lambda_1, \Lambda_2, \cdots, \Lambda_n) = \begin{bmatrix} \Lambda_1 & 0 & 0 & 0 \\ 0 & \Lambda_2 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Lambda_n \end{bmatrix} \tag{7}$$

where $\Lambda_1 \geq \Lambda_2 \geq \cdots \geq \Lambda_n$.

3. Wavelet Transformation for Inverse Acoustics

The basic way of implementing wavelet techniques into the boundary element method is schematically shown in Figure 1, and includes wavelet series expansions of the acoustic pressures on hologram surface and surface velocities on source surface, and direct matrix compression.

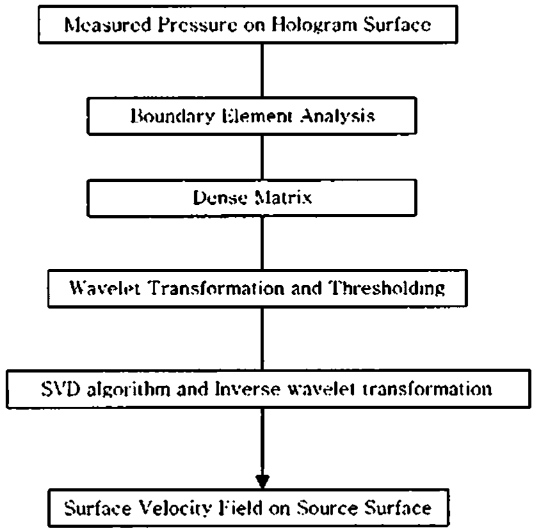

Fig. 1 Flow chart for BEM and FWT

The wavelet transformation can be applied as a post-processor to existing boundary element codes with minimal changes. The main reason is that wavelet compression techniques are only applied to the system matrix before the solution of the final system starts. Basically, this approach takes dense matrices and produces corresponding sparse ones. Therefore, all previous theory and numerical algorithms for the BEM can still be used, since the wavelet transform is directly applied to the matrices resulting from the original discretized matrix.

The Fast Wavelet Transform (FWT) algorithm proposed by Daubechies (1992) and Chui (1992), in its simplest form, takes a vector v of size N, with N a power of two, and produces another vector d of coefficients of the same size. As with many other discretizations, such as the discrete Fourier transform, the FWT can be represented by a square matrix W so that

$$Wv = d \tag{8}$$

where W is a sparse matrix whose elements are composed by shifts of a vector called a mask, particular to each wavelet family. The inverse matrix W^{-1} is often sparse and, if the wavelet family is orthogonal, then

$$WW^T = I \tag{9}$$

Using Eq. (3) and inserting identity matrix after each boundary element matrix, the following relation is obtained.

$$\begin{aligned} Wp_f &= W[G'']\left[\frac{\partial p}{\partial n}\right]_s \\ &= W[G'']\,W^T W\left[\frac{\partial p}{\partial n}\right]_s \end{aligned} \tag{10}$$

Letting $p_W = Wp_f$, $\left[\frac{\partial p}{\partial n}\right]_W = W\left[\frac{\partial p}{\partial n}\right]_s$ and $[G'']_W = W[G'']W^T$, Eq. (10) can be expressed as

$$p_W = [G'']_W\left[\frac{\partial p}{\partial n}\right]_W \tag{11}$$

Since $[G''] = U\Sigma V^H$, $[G'']_W$ has the following relationship with Σ, diagonal matrix composed of singular values,

$$
\begin{aligned}
[G'']_W &= W[G'']W^T = WU\Sigma V^H W^H \\
&= U_W \Sigma V_W^H
\end{aligned} \tag{12}
$$

where $U_W = WU$ and $V_W = WV$.

Equation (12) means that singular values of $[G'']$ are the same as those of $[G'']_W$. The new matrix $[G'']_W$, namely the two-dimensional wavelet transformed boundary element matrices, has only a few significant coefficients. And $[G'']_W$ can be effectively compressed by thresholding its elements. This operation is defined by, given a positive, real-valued, constant α,

$$
threshold(x) = \begin{cases} x & if\,|x| \geq \alpha \\ 0 & otherwise \end{cases} \tag{13}
$$

In this way, new sparse matrix $[\hat{G}'']_W$ is created. After thresholding, all of singular values with small values are set to be zero.

Fleischer et al. (1986) and Kim and Ih (1996) suggested the iterative solving method using SVD algorithm instead of direct inverse method or elimination method. The estimated surface velocity in Eq. (11) can be reconstructed from the measured pressure data by using the inverse iteration as

$$
\begin{aligned}
\left[\frac{\partial p}{\partial n}\right]_W^{l+1} &= \left[\frac{\partial p}{\partial n}\right]_W^{l} + \beta [G'']_W^H \left(p_W - [G'']_W \left[\frac{\partial p}{\partial n}\right]_W^{l}\right) \\
&= \beta [G'']_W^H p_W + (I - \beta [G'']_W^H [G'']_W) \left[\frac{\partial p}{\partial n}\right]_W^{l}
\end{aligned} \tag{14}
$$

where $\left[\frac{\partial p}{\partial n}\right]_W^{l}$ denotes the estimated source velocity at the l-th step, I is the identity matrix, and β is a convergence parameter. By the implementation of the iterative method, the estimated surface velocity in Eq. (14) can be written as

$$
\begin{aligned}
\left[\frac{\partial p}{\partial n}\right]_W^{l+1} &= \beta \sum_{i=0}^{l} (I - \beta [G'']_W^H [G'']_W) [G'']_W^H p_l \\
&= \beta \sum_{i=0}^{l} V_W \Sigma (I - \beta \Sigma^2)^i U_W^H p_l \\
&= V_W \begin{bmatrix} 1 - \dfrac{(1-\beta \Lambda_1^2)^{l-1}}{\Lambda_1} & 0 & \cdots & 0 \\ 0 & 1 - \dfrac{(1-\beta \Lambda_2^2)^{l-1}}{\Lambda_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 - \dfrac{(1-\beta \Lambda_i^2)^{l+1}}{\Lambda_i} \end{bmatrix}
\end{aligned} \tag{15}
$$

Finally, the surface velocity field in Eq. (5) can be easily calculated from the estimated source velocity by the use of the orthogonal property of the wavelet family in Eq. (9) such as

$$
\left[\frac{\partial p}{\partial n}\right]_S^{l+1} = W^T \left[\frac{\partial p}{\partial n}\right]_W^{l+1} \tag{16}
$$

Because Eqs. (15) and (16) can be considered as geometric series, the necessary and sufficient condition for convergence as the iteration number l is increased is given by

$$
|1 - \beta \Lambda_i^2| < 1, \text{ for } i = 1, 2, \cdots, n \tag{17}
$$

4. Numerical Example

In engineering applications, measured data have errors due to random fluctuation or system errors. Even if these errors can be reduced by averaging and calibration, the results are not free from errors. There are many error sources like bias, random errors due to microphone mismatching among microphones and position mismatching.

The goal is to show that wavelet transformation can be used for minimizing the noise impinged on acoustic field measured on a hologram surface and for reducing the computational cost. The quality of reconstructed velocity field should be judged because the field has inevitable errors due to numerical analysis. To judge the quality of reconstructed velocity field, the modal assurance criterion (MAC) is chosen because it is easy to compare the shapes.

$$
MAC = \frac{\sqrt{\sum_m \sum_n v_{exact(m,n)} v_{approximate(m,n)}^H \sum_m \sum_n v_{exact(m,n)}^H v_{approximate(m,n)}}}{\sqrt{\sum_m \sum_n v_{exact(m,n)}^H v_{approximate(m,n)}} \sqrt{\sum_m \sum_n v_{exact(m,n)}^H v_{approximate(m,n)}}} \tag{18}
$$

Another judging criterion is the peak-signal-to-noise ratio (PSNR) defined by

$$
PSNR = 20 \log_{10} \frac{255}{RMSE} \tag{19}
$$

where for an N by M image, $v_{(i,j)}$, the RMSE

(Root Mean Squared Error) is defined by

$$RSME = \sqrt{\frac{1}{N\cdot M}\sum_i\sum_j\{v_{exact(i,j)}v_{approximate(i,j)}\}^H\{v_{exact(i,j)}v_{approximate(i,j)}\}} \quad (20)$$

for complex variables v_{exact} and $v_{approximate}$. PSNR is used to evaluate objectively the image quality in image processing communities. The higher value of PSNR means better image reconstruction. The third criterion is L2 error e_2^Γ tha t defined as

$$e_2^\Gamma = \frac{\| v_{exact} - v_{approximate} \|_2}{\| v_{exact} \|_2} \quad (21)$$

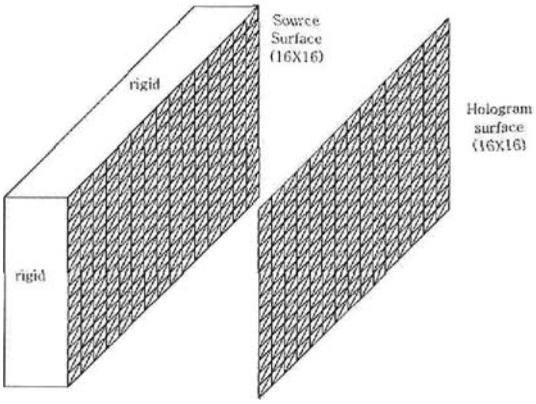

Fig. 2 Configuration of hologram surface and source surface

In this computer simulation, the wave number is considered as 0.116. The hologram surface is located at 0.3 m. The sampling spacing is 0.2 m. The number of hologram data is 16 by 16 as shown in Fig. 2. The simulation works have been done for three cases. The first one is an original image without any corruption. The second image is corrupted by Gaussian noise. The last one is same as the second one except de-noised by wavelet transformation and thresholding. And the backward propagation process has been applied to obtain the acoustic field image on the source plane. The acoustic images on the hologram sur-face and the velocity field on the source surface have been compared with the original that has one as its MAC value and infinite value (∞) as PSNR value and zero as L2 error value. The example has vibration mode corresponding to (3,2) mode as a simply supported radiator.

Two examples of robust NAH are studied with single vibration mode. Figures 3 and 4 show the real parts of transfer functions corresponding to uncompressed and compressed cases. Figure 3 denotes an original transfer function and Fig. 4 shows the compressed transfer function between hologram surface and source surface.

Figure 5 denotes an original image on the prediction surface (source surface), Fig. 6 shows the backward propagated image using the noisy

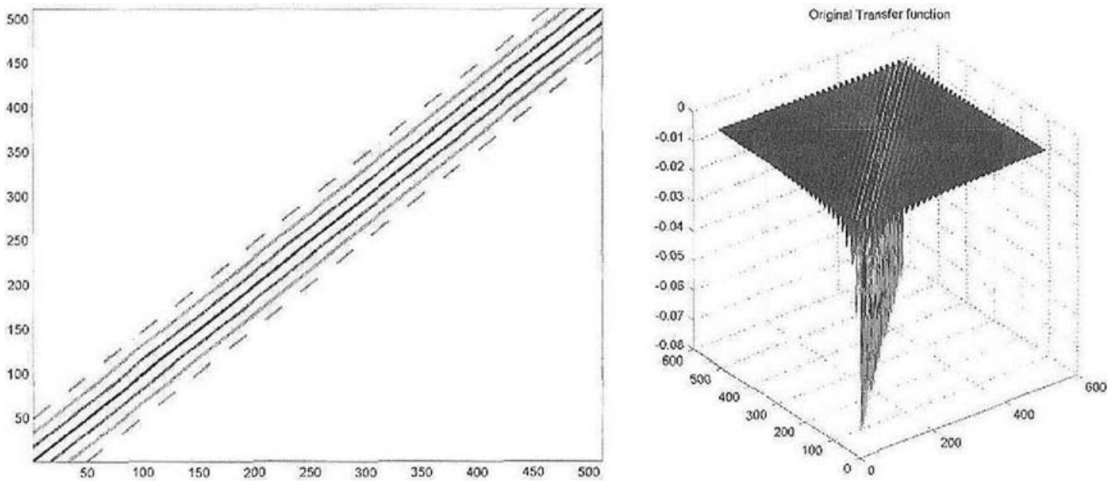

Fig. 3 Uncompressed image of transfor function

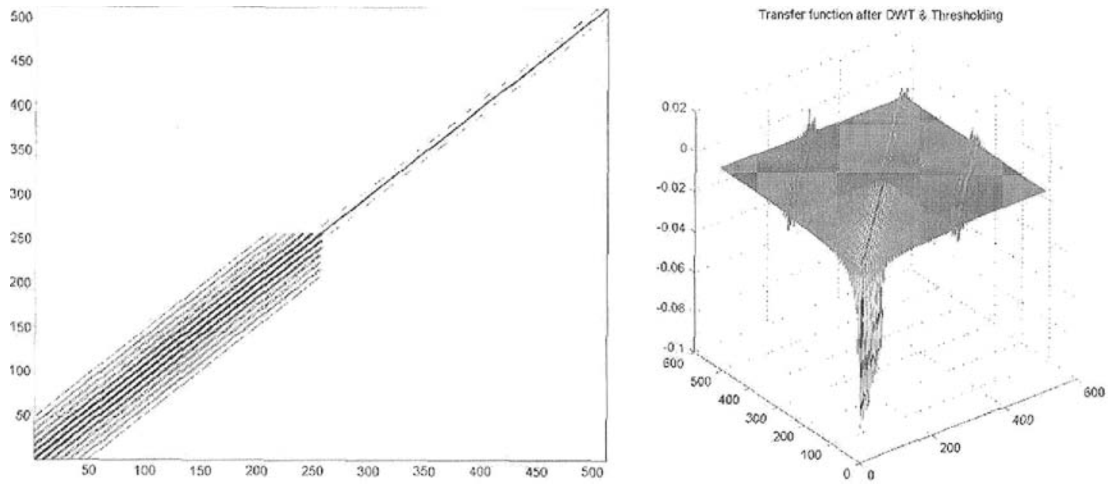

Fig. 4 Compressed Image of transfer function

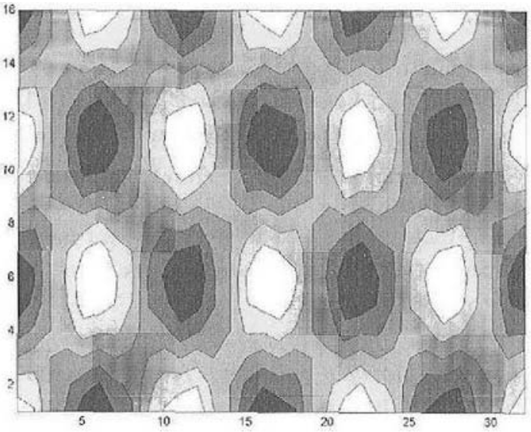

Fig. 5 Original surface velocity field

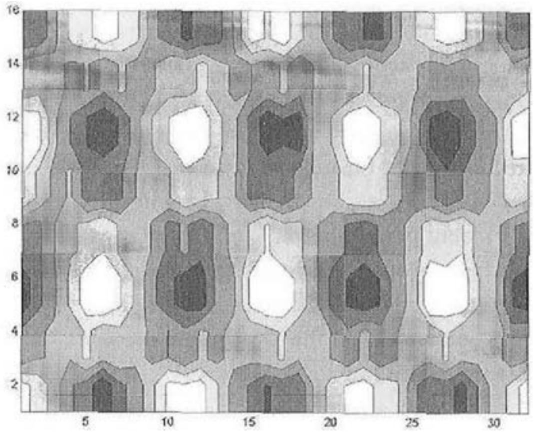

Fig. 7 Reconstructed surface velocity field using wavelet transformation

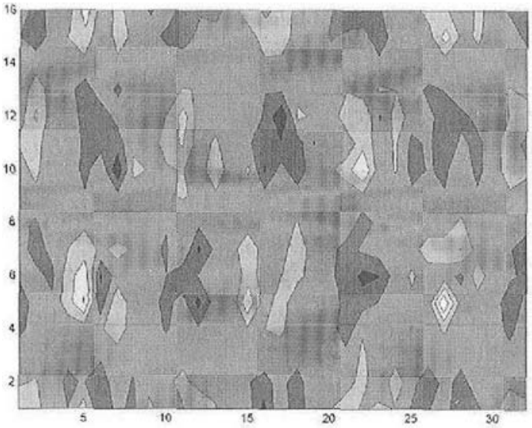

Fig. 6 Reconstructed surface velocity field without wavelet transformation

image and Fig. 7 represents the backward propagated image using the de-noised image by a wavelet transformation. MAC and PSNR values and L2 errors were calculated for every case for evaluating the quality of surface velocity fields. The results are shown in Table 1.

Based on the MAC, PSNR and L2 error values for judging quality, it is shown that the vibration field is enhanced by wavelet transformation scheme even if the acoustic field is corrupted by errors or noises on the hologram surface. Moreover, it is shown that impressive speed-up is obtained, with solution time being reduced to 14.5%.

Table 1 Several judgment criteria

Cases		MAC	PSNR	L2 error (%)	Solving Time (seconds)
Prediction Surface	No wavelet	0.6601	59.9729	28.0475	4.2360
	Wavelet	0.9407	80.8945	9.2864	0.6174

5. Conclusions

The NAH with the wavelet transformation is found to be appropriate to reconstruct a velocity field on the prediction surface even if the field on the hologram is noisy. This scheme is mainly concerned with inverse acoustics using NAH and wavelet analysis, that is, reconstruction of the velocity field on the noise source surface from measured acoustic pressure signals on the hologram surface. The proposed method obtains the enhanced velocity field on noise source plane reconstructed through the backward propagation of NAH. It is shown that the scheme is useful for identifying the velocity field on prediction surface through the computer simulation. And it is proven that the scheme is robust to high degrees of uncertainties in the measured data using wavelet transformation.

Acknowledgments

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